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Josephson-coupled layered superconductors with two order parameters: II. Lower critical magnetic field

E P Nakhmedov†§ and E V Tahirov‡

† Istanbul Technical University, Faculty of Sciences and Letters, Department of Physics, Maslak, 80626, Istanbul, Turkey

‡ Institute of Physics, Azerbaijan Academy of Sciences, 370143, H Cavid Street 33, Baku, Azerbaijan

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Abstract. A model for high-T_c oxides, consisting of Josephson-coupled superconducting layers with both intra- and interlayer pairings, is used to study the anomalous positive curvature in the temperature dependence of the lower critical magnetic field, $H_{c1}(T)$. An equilibrium distribution of magnetic fields in a single vortex is calculated for magnetic field parallel and perpendicular to the superconducting layers. By using this distribution the lower critical field H_{c1} is obtained. For Josephson-coupled layered superconductors with only intralayer pairing, the cross section of the single vortex parallel to the layers is known to have an elliptic form. The existence of two order parameters in our model gives rise to an additional anisotropy in the distribution of the magnetic field in a single vortex. The results obtained for H_{c1} make it possible to understand the experimentally observed anomalous enhancement of H_{c1} at low temperatures in $YBa_2Cu_3O_7$, as well as in Tl- and Bi-based superconductors.

1. Introduction

The DC magnetization measurements on high-T_c polycrystalline compounds [1, 2] and single crystals of $YBa_2Cu_3O_{7-x}$ [3, 4] show remarkable anomalous enhancements of the lower critical field, $H_{c1}(T)$, for both orientations of the magnetic field, parallel and perpendicular to the c axis. Non-conventional $H_{c1}(T)$ results have also been reported for bismuth-based ($Bi_{2.2}Sr_2Ca_{0.8}Cu_2O_{8+x}$) [5] and thallium-based ($Tl_2Ba_2CaCu_2O_x$) [6] superconductors. An unusual upturn of $H_{c1}(T)$ in $YBa_2Cu_3O_{7-x}$ single crystal with $T_c = 92$ K occurs for $T < T^* = 40$ K [3, 4]. For single-crystal $Tl_2Ba_2CaCu_2O_x$ with $T_c = 110$ K an anomalous enhancement of H_{c1} takes place below approximately 50–60 K [6].

Several theoretical models were put forward to explain this anomalous temperature dependence of the lower critical field, $H_{c1}(T)$, observed in high-T_c superconductors.

Koyama *et al* [7] have considered an extension of the Lawrence–Doniach model of multilayer structures, composed of arrays of superconducting (SC) and normal layers, to the case of superconductors with non-equivalent layers. The unusual temperature dependence of H_{c1} has been explained to be a result of rapid enhancement of the superconducting order parameter on the normal layers at low temperatures due to a proximity effect.

An essential deviation of the temperature dependence of H_{c1} from those of Bardeen–Cooper–Schrieffer (BCS) weak-coupling theory may be expected in the strong-coupling limit

§ Permanent address: Institute of Physics, Azerbaijan Academy of Sciences, 370143, H Cavid Street 33, Baku, Azerbaijan.

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of the Eliashberg theory [8, 9]. Low-frequency phonons, present in the high-T_c compounds, act as static impurities and determine the temperature dependence of the electron mean free path at low temperature. As a result, the shape of all the temperature dependence is changed on lowering the temperature [8, 9].

In this paper yet another mechanism is suggested to explain the positive curvature in the temperature dependence of both parallel and perpendicular components of H_{c1}. The existence of two order parameters in a superconductor is shown to lead to the anomalous enhancement of H_{c1} at low temperatures. For layered superconductors, pairing may occur both inside each layer and between neighbouring layers.

In the case of $t_\perp < kT_c^{(0)}$, where $t_\perp$ is the transverse resonance integral and $T_c^{(0)}$ is the critical temperature, formally evaluated by the mean-field method, the Josephson coupling between the layers is realized. The differential–difference Ginzburg–Landau free-energy functional for Josphson-coupled layered superconductors with two order parameters has been obtained in the previous paper [10].

In the present model intra- and interlayer pairing occur at two different critical temperatures, T_{c0} and T_{c1}. Such pairing behaviour was proposed *ab initio* by introducing two different constants for electron–electron attractive interactions. Though the mechanism of electron attraction is not specified in our model, an interlayer pairing may have non-phonon character, (see e.g. [11]). One possible mechanism of interlayer pairing is the polarization of dielectrics between SC layers. This mechanism was recently applied [12] to study layered high-temperature superconductors. The existence of two order parameters of pairing is expected to change the thermodynamic properties of the layered superconductors.

It should be noted that the lower critical magnetic field H_{c1} for Josephson-coupled layered superconductors with only intralayer pairing has been studied by many authors (see [13–16] and references therein) on the basis of the Lawrence–Doniach free-energy functional [17].

2. The magnetic field distribution in a single vortex

The vortical state is set up for type-II superconductors in a magnetic field, H, between two critical values, $H_{c1} < H < H_{c2}$. In this case the penetration of a magnetic field into the sample in the form of flux lines, each carrying a flux quantum of $\phi_0 = hc/2e$, is energetically favourable. In the vicinity of H_{c1} and $H \geqslant H_{c1}$, which implies a small vortex concentration, the interaction between vortices may be neglected. Then the lower critical field H_{c1} is obtained by the usual thermodynamic relation

$$H_{c1} = 4\pi\mathcal{E}_1/\phi_0$$

given that the energy $\mathcal{E}_1$ per unit length for a single vortex line is known.

For temperatures $(T_c - T)/T_c \gg t_\perp^2/T_c^2$ and magnetic fields slightly greater than H_{c1}, the influence of the field on the modulus of the order parameters $|\Delta_0|$ and $|\Delta_1|$ can be neglected, and we take $|\Delta_0| = \text{const}$ and $|\Delta_1| = \text{const}$. Then, representing Δ_k as

$$\Delta_k(r, j) = |\Delta_k| \exp[i\varphi_k(r, j)] \qquad (k = 0, 1)$$

with φ being the phases of the order parameters, the Ginzburg–Landau free-energy functional F with two order parameters, obtained in our previous work [10], may be rewritten as (see formula (23) in [10]):

$$F\{\varphi_0,\varphi_1\}=\sum_j\int \mathrm{d}^2r\left\{N_{s0}(T)\frac{\hbar^2}{8m}\left(\frac{\partial\varphi_0}{\partial \boldsymbol{r}}-\frac{2e}{\hbar c}\boldsymbol{A}\right)^2+N_{s1}(T)\frac{\hbar^2}{8m}\left(\frac{\partial\varphi_1}{\partial \boldsymbol{r}}-\frac{2e}{\hbar c}\boldsymbol{A}\right)^2\right.$$
$$+N_{s0}E_\perp\sum_{g=\pm1}\left[1-\cos\left(\varphi_0(\boldsymbol{r},j)-\varphi_0(\boldsymbol{r},j+g)+\frac{2ed}{\hbar c}gA_z\right)\right]$$
$$+N_{s1}\frac{E_\perp}{2}\sum_{g=\pm1}\left[1-\cos\left(\varphi_1(\boldsymbol{r},j)-\varphi_1(\boldsymbol{r},j+g)+\frac{2ed}{\hbar c}gA_z\right)\right]$$
$$-2E_{01}(N_{s0}N_{s1})^{1/2}\left[\cos\left(\varphi_1(\boldsymbol{r},j)-\varphi_0(\boldsymbol{r},j+1)+\frac{ed}{\hbar c}A_z\right)\right.$$
$$\left.\left.+\cos\left(\varphi_1(\boldsymbol{r},j)-\varphi_0(\boldsymbol{r},j)-\frac{ed}{\hbar c}A_z\right)\right]\right\}+\int\frac{H^2}{8\pi}\mathrm{d}V \tag{1}$$

Here, $N_{s0}(T)=2|\Delta_0|^2$ and $N_{s1}(T)=2|\Delta_1|^2$ are the densities of superconducting electrons for intra- and interlayer pairing, respectively. The remaining notations have the same meaning as in [10].

The temperature dependences of $N_{s0}(T)$ and $N_{s1}(T)$ are defined by equilibrium values of order parameters Δ_0 and Δ_1, which satisfy the Gor'kov equations (19) and (20) in [10]. The equilibrium values of Δ_0 and Δ_1 are studied in the appendix by minimizing the free-energy functional $F\{\Delta_0,\Delta_1\}$, presented by the expression (23) in [10].

The two components of the lower critical field H, parallel and perpendicular to the SC layers, will be studied separately. Let the magnetic field $\boldsymbol{H}$ be along the z-axis, which is normal to the layers, i.e. $\boldsymbol{H}=\{0,0,H\}$. The equations determining the equilibrium values of magnetic field and order-parameter phases are obtained by minimizing the free-energy functional (1) with respect to the components of the vector potential $\boldsymbol{A}$ and the order-parameter phases. For the sake of simplicity, we present here only the equations for the magnetic field:

$$-\frac{1}{4\pi}\frac{\partial H}{\partial y}=-\frac{\hbar^2}{4m}\frac{2e}{\hbar c}\sum_{i=0,1}N_{si}(T)\left(\frac{\partial\varphi_i(\boldsymbol{r},j)}{\partial x}-\frac{2e}{\hbar c}A_x\right) \tag{2}$$

$$\frac{1}{4\pi}\frac{\partial H}{\partial x}=-\frac{\hbar^2}{4m}\frac{2e}{\hbar c}\sum_{i=0,1}N_{si}(T)\left(\frac{\partial\varphi_i(\boldsymbol{r},j)}{\partial y}-\frac{2e}{\hbar c}A_y\right). \tag{3}$$

Equations (2) and (3) together with Maxwell's equation yield the well known London equation, namely,

$$\lambda_L^2\left(\partial^2H/\partial x^2+\partial^2H/\partial y^2\right)-H=0 \tag{4}$$

where λ_L is the London penetration depth along an SC layer, determined by the expression

$$\lambda_L^2=mc^2/\{4\pi e^2[N_{s0}(T)+N_{s1}(T)]\}. \tag{5}$$

For a single vortex centred at the origin, the solution of equation (4) for distances $|\boldsymbol{r}|\gg\xi_\parallel$ is known to be given (see e.g. [18]) as

$$H_{c1}^\perp=(\phi_0/4\pi\lambda_L^2)\left[\ln(\lambda_L/\xi_\parallel)+\mathcal{E}_0\right] \tag{6}$$

where $\xi_\parallel$ is the intralayer coherence length. The quantity $\mathcal{E}_0$ corresponds to the 'core' energy of the vortex filament, and $\mathcal{E}_0 \sim 1$.

According to the appendix, the densities of SC electrons $N_{s0}(T)$ and $N_{s1}(T)$ depend non-linearly on temperature and have positive curvature in the vicinity of the transition point. The same temperature dependence is also appropriate to the lower critical magnetic field $H_{c1}^{\perp}$.

For a magnetic field lying in SC layers and directed, say, parallel to the x-axis, i.e. $\boldsymbol{H} = \{H, 0, 0\}$, minimization of the free-energy functional (1) gives the following equations:

$$\begin{aligned}\frac{1}{4\pi}\frac{\partial H}{\partial y} = \frac{2ed}{\hbar c}E_\perp\Bigg[2N_{s0}\sin\left(\varphi_0(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j+1) + \frac{2ed}{\hbar c}A_z\right)\\ + N_{s1}\sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_1(\boldsymbol{r},j+1) + \frac{2ed}{\hbar c}A_z\right)\Bigg]\\ + 2E_{01}(N_{s0}N_{s1})^{1/2}\frac{ed}{\hbar c}\Bigg[\sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j+1) + \frac{ed}{\hbar c}A_z\right)\\ - \sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j) - \frac{ed}{\hbar c}A_z\right)\Bigg]\end{aligned} \tag{7}$$

$$-\frac{1}{4\pi}\frac{\partial H}{\partial z} = -\frac{e\hbar}{2mc}\sum_{i=0,1}N_{si}(T)\left(\frac{\partial\varphi_i(\boldsymbol{r},j)}{\partial y} - \frac{2e}{\hbar c}A_y\right) \tag{8}$$

$$\begin{aligned}-\frac{\hbar^2}{4m}\frac{\partial^2\varphi_0}{\partial \boldsymbol{r}^2} + 2E_\perp\Bigg[\sin\left(\varphi_0(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j+1) + \frac{2ed}{\hbar c}A_z\right)\\ + \sin\left(\varphi_0(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j-1) - \frac{2ed}{\hbar c}A_z\right)\Bigg]\\ - 2E_{01}\left(\frac{N_{s1}}{N_{s0}}\right)^{1/2}\Bigg[\sin\left(\varphi_1(\boldsymbol{r},j-1) - \varphi_0(\boldsymbol{r},j) + \frac{ed}{\hbar c}A_z\right)\\ + \sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j) - \frac{ed}{\hbar c}A_z\right)\Bigg] = 0\end{aligned} \tag{9}$$

$$\begin{aligned}-\frac{\hbar^2}{4m}\frac{\partial^2\varphi_1}{\partial \boldsymbol{r}^2} + E_\perp\Bigg[\sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_1(\boldsymbol{r},j+1) + \frac{2ed}{\hbar c}A_z\right)\\ + \sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_1(\boldsymbol{r},j-1) - \frac{2ed}{\hbar c}A_z\right)\Bigg]\\ + 2E_{01}\left(\frac{N_{s0}}{N_{s1}}\right)^{1/2}\Bigg[\sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j+1) + \frac{ed}{\hbar c}A_z\right)\\ + \sin\left(\varphi_1(\boldsymbol{r},j) - \varphi_0(\boldsymbol{r},j) - \frac{ed}{\hbar c}A_z\right)\Bigg] = 0.\end{aligned} \tag{10}$$

In principle, we can obtain an equation for the magnetic field by eliminating the phases φ_0 and φ_1 from (7) and (8), and replacing vector potential $\boldsymbol{A}$ by H by using Maxwell's equations. Since (7)–(10) are non-linear equations, the elimination of φ_0 and φ_1 is carried out after expansion of sine functions in (7)–(10). Transforming the discrete variable j into the continuous variable $z = jd$ and replacing the finite differences by differentiations in

(7)–(10), we get the following equations for the magnetic field H and the phases φ_0 and φ_1:

$$\lambda_J^2\frac{\partial^2 H}{\partial y^2}+\lambda_L^2\frac{\partial^2 H}{\partial z^2}-H=-\frac{\hbar c}{2e}\frac{N_{s1}}{N_{s0}+N_{s1}}\left(1-\frac{4md^2E_\perp}{\hbar^2}\frac{\lambda_J^2}{\lambda_L^2}\right)\frac{\partial^2}{\partial z\partial y}[\varphi_0(\boldsymbol{r},j)-\varphi_1(\boldsymbol{r},j)] \tag{11}$$

$$\frac{\partial^2\varphi_0}{\partial y^2}+\frac{8E_\perp md^2}{\hbar^2}\frac{\partial^2\varphi_0}{\partial z^2}-\frac{16mE_{01}}{\hbar^2}\left(\frac{N_{s1}}{N_{s0}}\right)^{1/2}(\varphi_0-\varphi_1)=0 \tag{12}$$

$$\frac{\partial^2\varphi_1}{\partial y^2}+\frac{4E_\perp md^2}{\hbar^2}\frac{\partial^2\varphi_1}{\partial z^2}-\frac{16mE_{01}}{\hbar^2}\left(\frac{N_{s0}}{N_{s1}}\right)^{1/2}(\varphi_1-\varphi_0)=0 \tag{13}$$

where λ_L is the London penetration depth along SC layers, defined by expression (5), and λ_J in (11) is the penetration depth in the z direction, defined as

$$\lambda_J^{-2}=(16\pi e^2d^2/\hbar^2c^2)[2E_\perp N_{s0}+E_\perp N_{s1}+E_{01}(N_{s0}N_{s1})^{1/2}]. \tag{14}$$

Since λ_J^{-2} contains both densities, N_{s0} and N_{s1}, of superconducting electrons, its temperature dependence must reveal similar anomalies as λ_L^{-2}. Equation (11) gives the distribution of magnetic field in a single vortex, directed parallel to SC layers, under the boundary condition requiring the total magnetic field flux through the yz plane to be equal to the flux quantum ϕ_0. As can be seen from (11), the existence of two order parameters reduces the equation for H to a non-homogeneous equation. For $N_{s0}\to 0$ (or $N_{s1}\to 0$) the right-hand side of (11) vanishes. The solutions of (12) and (13) are:

$$\varphi_0(y,z)=\tan^{-1}\left[\frac{y}{z}\left(\frac{8E_\perp md^2}{\hbar^2}\right)^{1/2}\right] \tag{15a}$$

$$\varphi_1(y,z)=\tan^{-1}\left[\frac{y}{z}\left(\frac{4E_\perp md^2}{\hbar^2}\right)^{1/2}\right]. \tag{15b}$$

After the coordinate transformation

$$y=\lambda_J\rho\sin\theta \qquad \text{and} \qquad z=\lambda_L\rho\cos\theta$$

equation (11) takes the following form:

$$\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial H}{\partial\rho}\right)+\frac{\partial^2 H}{\partial\theta^2}-\rho^2H=f(\theta) \tag{16}$$

where

$$f(\theta)=\beta_0\frac{\beta_0^2\sin^2\theta-\cos^2\theta}{(\cos^2\theta+\beta_0^2\sin^2\theta)^2}+\beta_1\frac{\beta_1^2\sin^2\theta-\cos^2\theta}{(\cos^2\theta+\beta_1^2\sin^2\theta)^2} \tag{17}$$

with

$$\beta_0=\frac{\lambda_J}{\lambda_L}\left(\frac{8E_\perp md^2}{\hbar^2}\right)^{1/2} \qquad \text{and} \qquad \beta_1=\frac{\lambda_J}{\lambda_L}\left(\frac{4E_\perp md^2}{\hbar^2}\right)^{1/2}. \tag{18}$$

When the magnetic field is parallel to the layers, the study of a vortex in a Josephson-coupled layered SC with only intralayer pairing has revealed a non-symmetric distribution of magnetic field in a vortex [13–16]. The cross section has been found to be in an elliptic form. In our model, the existence of the second component of the order parameter produces an additional angular dependence of magnetic field in a vortex.

The solution of equation (16) is chosen as

$$H(\rho,\theta)=\sum_{n=0}^{\infty}R_n(\rho)\cos(n\theta). \tag{19}$$

By representing also the right-hand side of (16) in the form

$$f(\theta)=\sum_{n=0}^{\infty}f_n\cos(n\theta) \tag{20}$$

with

$$f(\theta)=\frac{2}{\pi}\int_0^{\pi}f(\theta)\cos(n\theta)\,\mathrm{d}\theta \tag{21}$$

the angular dependence of (16) may be separated. The equation for $R_n(\rho)$ is a non-homogeneous Bessel equation. After solving this equation we can represent $H(\rho,\theta)$ as

$$\begin{aligned}H(\rho,\theta)=\frac{\phi_0}{2\pi\lambda_L\lambda_J}\bigg[K_0(\rho)+\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\varphi\, f(\varphi)\bigg(\int_\rho^{\infty}\frac{\mathrm{d}x}{x}K_0([\rho^2+x^2+2\rho x\cos(\varphi+\theta)]^{1/2})\\ -\int_0^{\rho}\frac{\mathrm{d}x}{x}K_0([\rho^2+x^2+2\rho x\cos(\varphi+\theta)]^{1/2})\bigg)\bigg]\end{aligned} \tag{22}$$

where K_n is the modified Bessel function. Approximate calculations of the integrals in (22) for $\rho \ll 1$ yield the following expression for $H(\rho,\theta)$:

$$H(\rho,\theta)=\frac{\phi_0}{2\pi\lambda_L\lambda_J}[\ln(1/\rho)+\tfrac{1}{2}\beta\rho^2\cos(2\theta)+O(\rho^4\cos(4\theta))] \tag{23}$$

where

$$\beta=\beta_0/(1+\beta_0)^2+\beta_1/(1+\beta_1)^2. \tag{24}$$

In cartesian coordinates the expression (23) looks like

$$H(y,z)=(\phi_0/4\pi\lambda_L\lambda_J)\{-\ln[(y/\lambda_J)^2+(z/\lambda_L)^2]+\beta[(z/\lambda_L)^2-(y/\lambda_J)^2]\}\qquad y\ll\lambda_J,\ z\ll\lambda_L. \tag{25}$$

Using expression (25) for $H(y,z)$, we can calculate the energy $\mathcal{F}$ of the vortex filament. Since the centre of the vortex lies between the layers for a magnetic field directed parallel to the layers, the lower limit of integration in $\mathcal{F}$ with respect to z must be equal to d. As a result, the calculations give the following expression for $H_{c1}^{\parallel}$:

$$H_{c1}^{\parallel}=(\phi_0/4\pi\lambda_L\lambda_J)[\ln(\lambda_L/d)+(2-4/\pi)\beta]. \tag{26}$$

An unusual temperature dependence of the penetration depth λ_L and λ_J (see (5) and (14)) gives rise to the existence of a positive curvature in the temperature dependence of $H_{c1}^{\parallel}$.

3. Conclusions

In this paper we have studied the equilibrium distribution of the magnetic field in a single magnetic vortex for a Josephson-coupled layered superconductor with two order parameters, corresponding to inter- and intralayer pairings. The lower critical magnetic field was also calculated for the system under consideration.

For a magnetic field parallel to SC layers, the equilibrium distribution of the field in the cross section of a single vortex reveals an additional dependence on the direction (see equations (11), (16) and (23), (25)) due to the existence of two components of the order parameter.

Starting from the result obtained, it may be possible to understand the anomalous upturn of $H_{c1}(T)$ in high-T_c oxides [1–6] for an arbitrarily oriented magnetic field.

The anisotropy of the magnetic properties of the model studied here is described by two components of the penetration depth, λ_L and λ_J (see equations (5) and (14)). The anisotropy of the lower critical magnetic field represented as $H_{c1}^{\perp}/H_{c1}^{\parallel}$ is mostly characterized by the ratio λ_J/λ_L, i.e.

$$\frac{H_{c1}^{\perp}}{H_{c1}^{\parallel}} = \frac{\lambda_J}{\lambda_L}\frac{\ln(\lambda_L/\xi_\parallel)}{\ln(\lambda_L/d)}. \tag{27}$$

For the known high-T_c materials the value of this anisotropy is rather large. For instance, in $YBa_2Cu_3O_7$ single crystals, ratio λ_J/λ_L has an approximate value of 5, i.e. $\lambda_J/\lambda_L \sim 5$ [3]. In Bi- and Tl-based oxides this ratio is larger but not very much so.

By using the formulae (5) and (14) we obtain

$$\frac{\lambda_J^2}{\lambda_L^2} = \frac{\hbar^2}{4md^2}\frac{N_{s0}+N_{s1}}{2E_\perp N_{s0} + E_\perp N_{s1} + E_{01}(N_{s0}N_{s1})^{1/2}}. \tag{28}$$

The Josephson energy, $E_\perp$, in (28) depends on the characteristic small parameter of the electron anisotropy $t_\perp/\mathcal{E}_F$ as

$$E_\perp = t_\perp^2/32\mathcal{E}_F.$$

Here $t_\perp$ is the resonance integral between layers and $\mathcal{E}_F$ is the Fermi energy of an electron inside layers. The second parameter E_{01} in (28), which is due to the existence of both intra- and interlayer pairings, depends linearly on the anisotropy parameter $t_\perp/\mathcal{E}_F$ (see [10]):

$$E_{01} = \frac{1}{2\sqrt{2}}\left(\frac{kT}{\mathcal{E}_F}\right)^2 t_\perp\left[\frac{1}{1-\mu_\parallel}\tanh\left(\frac{\mathcal{E}_F}{2kT}(1-\mu_\parallel)\right) - \tanh\left(\frac{\mathcal{E}_F}{2kT}\right)\right].$$

For the case of decreasing $t_\perp$, the last term in the denominator of (28) may be the leading term. A crossover from the dependence given as

$$\frac{H_{c1}^{\perp}}{H_{c1}^{\parallel}} \sim \frac{\hbar^2}{md^2}\frac{\mathcal{E}_F}{t_\perp^2}$$

to a dependence given as

$$\frac{H_{c1}^{\perp}}{H_{c1}^{\parallel}} \sim \frac{\hbar^2}{2md^2}\left(\frac{\mathcal{E}_F}{kT}\right)^2\frac{1}{t_\perp}$$

is expected with increasing anisotropy of superconductors. Therefore, a considerable increase of the anisotropy of a system may result in a weak increase in the anisotropy of magnetic properties of the system.

In this work we have not studied the influence of thermal fluctuations of the magnetic vortices. However, in high-T_c oxides an isolated vortex line along the c-axis can wander significantly in the ab plane due to the high critical temperatures and small coherence length [19–21]. This effect may lead to an entanglement of flux lines and, as a result, to a new entangled flux liquid phase in a magnetic field $H \geqslant H_{c1}$ [19, 20].

The effect of fluctuations on the properties of Josephson-coupled layered superconductors with only intralayer pairing in a magnetic field parallel to the layers was considered by Efetov [22]. The cores of the vortices for a magnetic field parallel to the layers fitted between the superconducting layers. Motion of these vortices in a direction normal to the superconducting planes is difficult owing to the existence of Peierls friction [23]. Since the correlation length inside CuO_2 planes in high-T_c superconductors is shorter than it is in conventional superconductors, entanglement of flux lines seems to occur in the direction parallel to the superconducting planes, i.e. these vortex lines may wander only on the superconducting layers.

Appendix

The temperature dependences of SC electron concentrations, $N_{s0}(T) = 2|\Delta_0|^2$, and $N_{s1}(T) = 2|\Delta_1|^2$, are defined by the equilibrium values of the order parameters, which are found by solving Gor'kov's equations presented by expressions of (19) and (20) in [10]. We shall again minimize the free-energy functional of (23) with (A6) in [10] over Δ_0 and Δ_1 to keep the convenient notations. In the main approximation ($\beta_1, \beta_2 \ll \beta_0$) we obtain

$$\alpha_0(T)\Delta_0^* + 4E_{01}\Delta_1^* + \beta_0\left(|\Delta_0|^2 + |\Delta_1|^2\right)\Delta_0^* = 0 \tag{A1}$$

$$\alpha_1(T)\Delta_1^* + 4E_{01}\Delta_0^* + \beta_0\left(\tfrac{1}{2}|\Delta_0|^2 + |\Delta_1|^2\right)\Delta_1^* = 0. \tag{A2}$$

For half-filling ($E_{01} = 0$) these equations have the following solutions (we assume $T_{c0} > T_{c1}$):

(i) $T > T_{c0}$:

$$\Delta_0^* = 0 \qquad \Delta_1^* = 0 \tag{A3}$$

(ii) $T_{c1} < T < T_{c0}$:

$$\Delta_0^* \neq 0 \qquad \Delta_1^* = 0$$

$$|\Delta_0|^2 = -\frac{\alpha_0}{\beta_0} = -\frac{p_F^2}{\pi\hbar^2}\ln\left(\frac{T}{T_{c0}}\right) \simeq \frac{p_F^2}{\pi\hbar^2}\frac{T_{c0} - T}{T_{c0}}. \tag{A4}$$

(iii) $T_{c1}^2/T_{c0} < T < T_{c1}$:

$$\Delta_0^* = 0 \qquad \Delta_1^* \neq 0$$

$$|\Delta_1|^2 = -\frac{2\alpha_1}{\beta_0} = -\frac{2p_F^2}{\pi\hbar^2}\ln\left(\frac{T}{T_{c1}}\right) \simeq \frac{2p_F^2}{\pi\hbar^2}\frac{T_{c1} - T}{T_{c1}}. \tag{A5}$$

(iv) $T < T_{c1}^2/T_{c0}$:

$$\Delta_0^* \neq 0 \qquad \Delta_1^* \neq 0$$

and in this interval $|\Delta_0|^2$ and $|\Delta_1|^2$ are obtained as a solution of the following system of equations:

$$\alpha_0 + \beta_0|\Delta_0|^2 + \beta_0|\Delta_1|^2 = 0 \tag{A6}$$

$$\alpha_1 + \beta_0|\Delta_0|^2 + \tfrac{1}{2}\beta_0|\Delta_1|^2 = 0 \tag{A7}$$

$$|\Delta_0|^2 = \frac{\alpha_0 - 2\alpha_1}{\beta_0} = \frac{p_F^2}{\pi\hbar^2}\ln\left(\frac{T_{c1}^2}{TT_{c0}}\right) \simeq \frac{p_F^2}{\pi\hbar^2}\left(1 - \frac{2T_{c0} - T_{c1}}{T_{c0}T_{c1}}T\right) \tag{A8}$$

$$|\Delta_1|^2 = \frac{2(\alpha_1 - \alpha_0)}{\beta_0} = \frac{2p_F^2}{\pi\hbar^2}\ln\left(\frac{T_{c0}}{T_{c1}}\right). \tag{A9}$$

Thus, for the half-filling case the temperature dependence of SC electron concentrations $N_{s0}(T) = 2|\Delta_0|^2$ and $N_{s1} = 2|\Delta_1|^2$ are defined by equations (A3)–(A9). The transition at $T = T_{c1}$ seems to be a first-order phase transition with concentration jump

$$\delta N_s = \frac{p_F^2}{\pi\hbar^2}\frac{T_{c0} - T_{c1}}{T_{c0}}.$$

It should be noted that the equations (A3)–(A9) give positive curvature in the temperature dependence of the SC electron concentration because of the different slopes in the temperature dependences of (A4) and (A5).

For a case of $E_{01} \neq 0$ the temperature dependences of $N_{s0}(T)$ and $N_{s1}(T)$ become rather complicated. Mixing of the equations (A1) and (A2) due to the linear terms ($\sim E_{01}$) turns out to be essential. So, equations (A1) and (A2) take the following forms:

$$[\alpha_0(T) + \beta_0|\Delta_0|^2]\Delta_0^* + 4E_{01}\Delta_1^* = 0 \tag{A10}$$

$$4E_{01}\Delta_0^* + [\alpha_1(T) + \tfrac{1}{2}|\Delta_1|^2]\Delta_1^* = 0. \tag{A11}$$

These equations may be reduced to the uncoupled ones:

$$\left\{(4E_{01})^2[\alpha_0(T)\alpha_1(T) - (4E_{01})^2] + \beta_0\left[\tfrac{1}{2}\alpha_0^3 + \alpha_1(4E_{01})^2\right]|\Delta_0|^2 + \tfrac{3}{2}\beta_0^2\alpha_0^2|\Delta_0|^4\right\}\Delta_0^* = 0 \tag{A12}$$

$$\left\{(4E_{01})^2[\alpha_0(T)\alpha_1(T) - (4E_{01})^2] + \beta_0\left[\alpha_1^3 + \tfrac{1}{2}\alpha_0(4E_{01})^2\right]|\Delta_1|^2 + \tfrac{3}{2}\beta_0^2\alpha_1^2|\Delta_1|^4\right\}\Delta_1^* = 0. \tag{A13}$$

The first terms of equations (A12) and (A13)

$$(4E_{01})^2[\alpha_0(T)\alpha_1(T) - (4E_{01})^2] = (16E_{01}/\mathcal{E}_F)^2 T_{c0}T_{c1}(T - T_{c0}^*)(T - T_{c1}^*) \tag{A14}$$

vanish at the new critical temperatures T_{c0}^* and T_{c1}^*, defined as

$$T_{ci}^* = \tfrac{1}{2}\left\{(T_{c0} + T_{c1}) \pm \left[(T_{c0} - T_{c1})^2 + (2\mathcal{E}_F E_{01})^2/T_{c0}T_{c1}\right]^{1/2}\right\}. \tag{A15}$$

Here the upper sign on the RHS corresponds to T^*_{c0}. Since $T_{c0} > T_{c1}$, always $T^*_{c0} > T_{c0}$ and $T^*_{c1} < T_{c1}$.

As is seen from equations (A12) and (A13), $\Delta^*_0 = \Delta^*_1 = 0$ at $T > T^*_{c0}$ and $\Delta^*_0 \neq 0$, $\Delta^*_1 \neq 0$ at $T < T^*_{c0}$ (according to equations (A10) and (A11) the condition $\Delta_0 \neq 0$ requires $\Delta_1 \neq 0$ and vice versa). Thus the transition to the SC state occurs at relatively high temperature $T^*_{c0} > T_{c0}, T_{c1}$. At $T_{c0} < T < T^*_{c0}$ and $T < T^*_{c1}$ the approximate solutions of (A12) and (A13) are

$$|\Delta_0|^2 = -\frac{(4E_{01})^2}{\beta_0}\frac{\alpha_0(T)\alpha_1(T) - (4E_{01})^2}{\left[\frac{1}{2}\alpha_0 + \alpha_1(4E_{01})^2\right]} \tag{A16}$$

$$|\Delta_1|^2 = -\frac{(4E_{01})^2}{\beta_0}\frac{\alpha_0(T)\alpha_1(T) - (4E_{01})^2}{\left[\alpha_1^3 + \frac{1}{2}\alpha_0(4E_{01})^2\right]}. \tag{A17}$$

In $T_{c1} < T < T_{c0}$ and $T^*_{c1} < T < T_{c1}$ the equilibrium values of $|\Delta_0|^2$ and $|\Delta_1|^2$ are generally defined by positive roots of (A12) and (A13).

As is seen from (A16) and (A17), in the temperature interval $T_{c0} < T < T^*_{c0}$ (where $\alpha_0\alpha_1 < (4E_{01})^2$), $|\Delta_0|^2$ and $|\Delta_1|^2$ are monotonic functions of T with positive second derivatives, which means the existence of positive curvature in the temperature dependence of both SC electron densities, $|\Delta_0|^2$ and $|\Delta_1|^2$.

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